2024-2025
Fall Semester

Course of Power System Analysis

Transformers

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Outline

The ideal transformer model

Magnetically coupled coils

The equivalent circuit of a single-phase transformer

Simplified equivalent circuit of single phase transformer

Three-phase transformers

The ideal transformer model
Hypothesis

a) Flux varies **sinusoidally** in the core

+

<u>Ideal transformer:</u>

b) **Permeability** μ in the core **is infinite**;

c) **The flux is entirely confined into the core** and links all of the turns of both coils (or windings)

d) **Core losses are zero** (electrical conductivity of the magnetic core is zero)

e) **Winding resistances are zero** (Electrical conductivity of the windings is infinite)

A transformer is a static electrical machine made by of two (or more) coils (or windings) placed so that they are linked by the same magnetic flux.

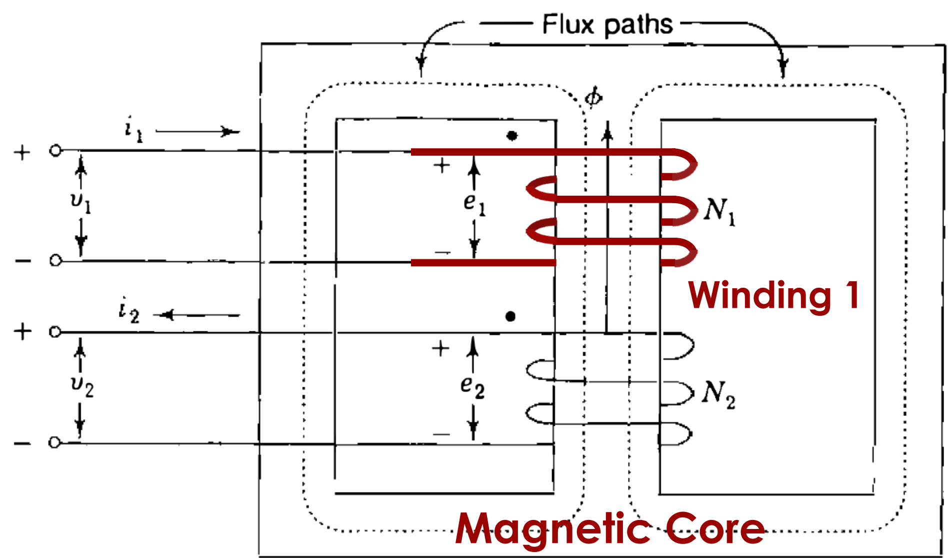


Figure 1.1 – Two-winding transformer.

The ideal transformer model

- Hypothesis **(b), (c)** and **(d)** → The flux ϕ that links winding 1 is the same that links winding 2 (i.e., there is **no leakage of the flux**)
- Winding resistances are zero **(e)** → **the voltages $\boldsymbol{e_1}$ and $\boldsymbol{e_2}$ induced by the changing flux must equal the terminal voltages $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$** (Faraday's law).

$$v_1 = e_1 = N_1 \frac{d\phi}{dt} \tag{1.1}$$

$$v_2 = e_2 = N_2 \frac{d\phi}{dt} \tag{1.2}$$

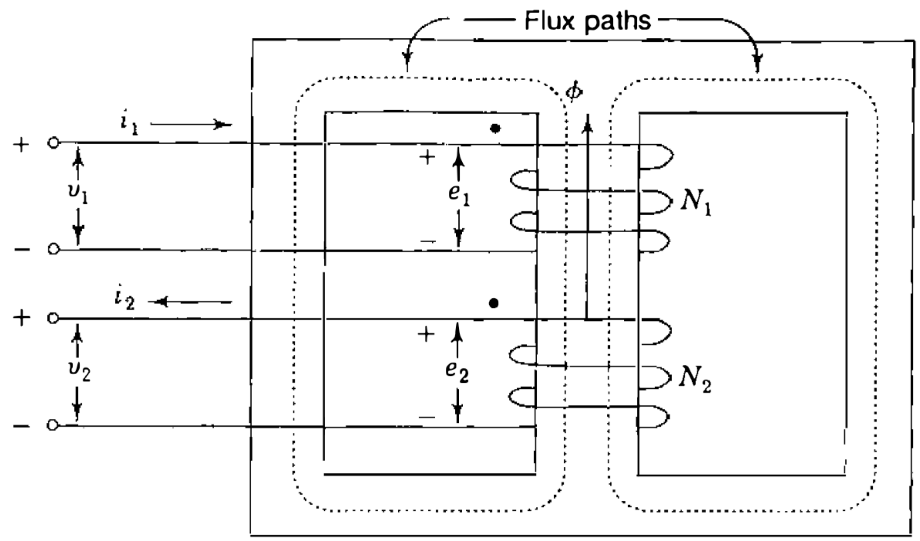


Figure 1.1 – Two-winding transformer.

The ideal transformer model

- Flux varies sinusoidally in the core **(a)** → phasor representation of voltages:

$$v_1 \xrightarrow{j\omega} \bar{V}_1, v_2 \xrightarrow{j\omega} \bar{V}_2, e_1 \xrightarrow{j\omega} \bar{E}_1, e_2 \xrightarrow{j\omega} \bar{E}_2$$

- Dividing Eq. (1.1) by (1.2) written in the frequency domain, we obtain:

$$\boxed{\frac{|\bar{V}_1|}{|\bar{V}_2|} = \frac{|\bar{E}_1|}{|\bar{E}_2|} = \frac{\mathrm{N}_1}{\mathrm{N}_2}} \qquad (1.3)$$

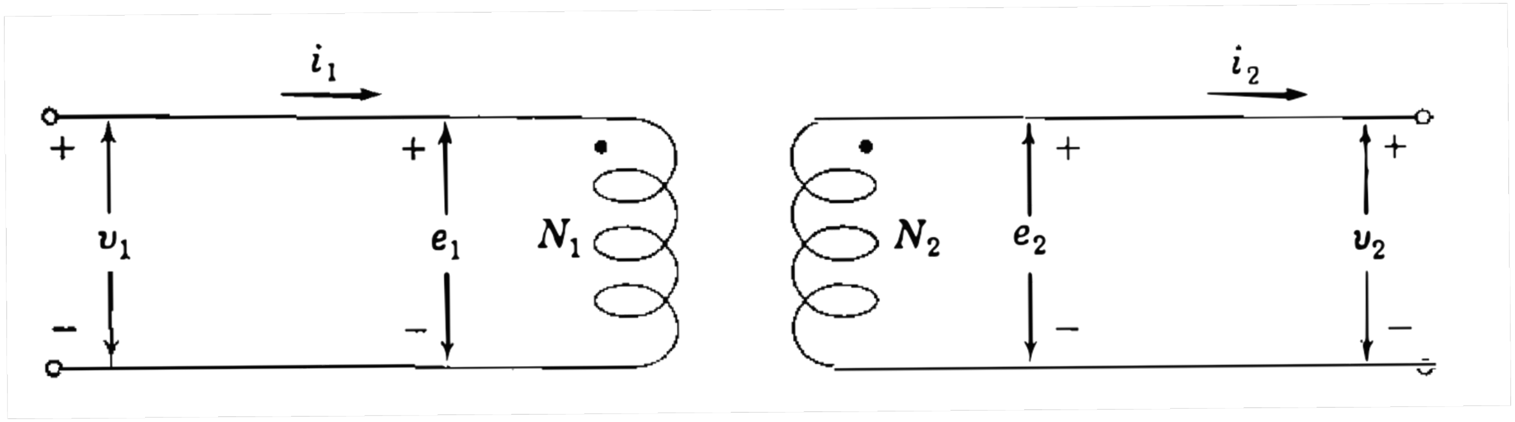


Figure 1.2 – Schematics representation of an **ideal** two winding transformer

The ideal transformer model

- By applying Ampère's law $\left(\oint_{\ell} H \cdot dl = \iint_{\mathcal{S}} J \cdot dS\right)$, where H is the magnetic field strength and J is the current density, around each of the closed paths ℓ of flux shown by dotted lines in Fig. 1.1 and the corresponding surface $\mathcal{S}$ we obtain:

$$\oint H \cdot dl = N_1 i_1 - N_2 i_2 \tag{1.4}$$

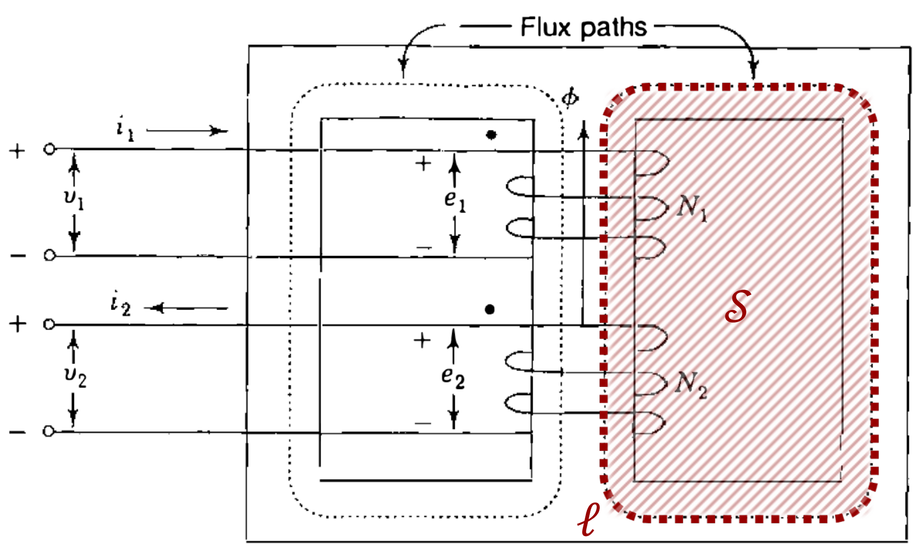


Figure 1.1 – Two-winding transformer.

The ideal transformer model

- Thanks to the hypothesis of infinite permeability (b), it is possible to say that $\oint_{\ell} H \cdot dl = 0$. If this were not true, flux density (being equal to μH) would be infinite. As a consequence, converting the currents to phasor form Eq. 1.4 becomes:

$$0 = N_1 \bar{I}_1 - N_2 \bar{I}_2 \tag{1.5}$$

$$\boxed{\frac{|\bar{I}_1|}{|\bar{I}_2|} = \frac{N_2}{N_1}} \tag{1.6}$$

Note that I_1 and I_2 are in phase if we choose the current to be like in Figure 1.1, or 1.2. If the direction chosen for either current is reversed, they are 180° out of phase.

The ideal transformer model

- The transformer winding across which an impedance or another load may be connected is called the **secondary winding**, and any circuit elements connected to this winding are said to be on the **secondary side* of the transformer.**
- The winding which is toward the source of energy is called the **primary winding** on the **primary side***.
- If an impedance Z_2 is connected across winding 2 of Figs. 1.1 or 1.2 we get:

$$\bar{Z}_2 = \frac{\bar{V}_2}{\bar{I}_2} \tag{1.7}$$

- By substituting for V_2 and I_2 the values found in Eqs. (1.3) and (1.6)

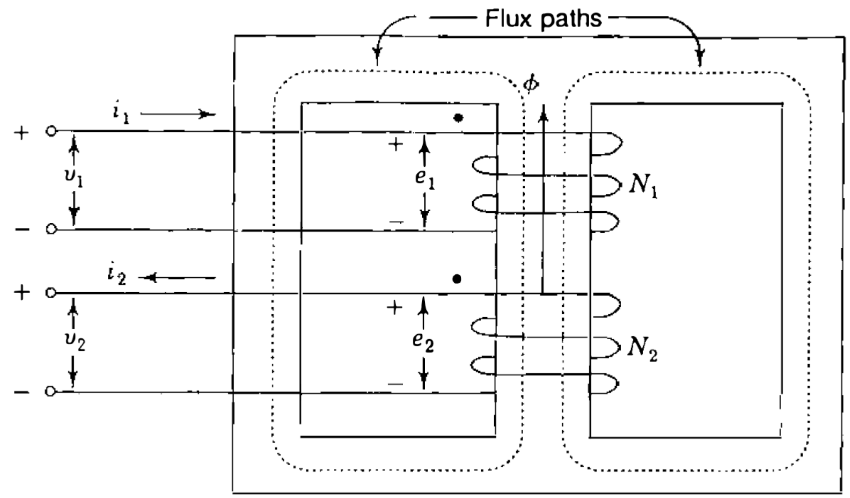
$$\bar{Z}_2 = \frac{\left({}^{N_2}/_{N_1}\right)\bar{V}_1}{\left({}^{N_1}/_{N_2}\right)\bar{I}_1} = \left(\frac{N_2}{N_1}\right)^2 \frac{\bar{V}_1}{\bar{I}_1} = \left(\frac{N_2}{N_1}\right)^2 \bar{Z}_2' \tag{1.8}$$

Where $\bar{Z}_2'$ is the equivalent impedance seen by the primary winding $\bar{Z}_2' = \frac{\bar{V}_1}{\bar{I}_1}$

** In the power system energy often will flow in either direction through a transformer and the designation of primary and secondary loses its meaning.*

The ideal transformer model

Summary



Hypothesis:

- Flux varies **sinusoidally** in the core
- **Ideal transformer**

Main equations:

$$\boxed{\frac{|\bar{V}_1|}{|\bar{V}_2|} = \frac{N_1}{N_2}} \quad (1.3)$$

$$\boxed{\frac{|\bar{I}_1|}{|\bar{I}_2|} = \frac{N_2}{N_1}} \quad (1.6)$$

Power balance: it is also worth noting that the complex power input to the primary winding equals the complex power output from the secondary winding since we are considering an ideal transformer. In fact:

$$\bar{S}_1 = \bar{V}_1 \underline{I}_1 = \frac{N_1}{N_2}\bar{V}_2 \cdot \frac{N_2}{N_1}\underline{I}_2 = \bar{V}_2 \underline{I}_2 = \bar{S}_2 \quad (1.9)$$

Outline

The ideal transformer model

Magnetically coupled coils

The equivalent circuit of a single-phase transformer

Simplified equivalent circuit of single phase transformer

Three-phase transformers

Magnetically coupled coils

Hypothesis

Ideal transformer:

Permeability μ in the core **is infinite**;

The flux is confined to the core and links all of the turns of both windings

Core losses are zero (Electrical conductivity of the core is zero)

Winding resistances are zero

STUDY CASE:

Permeability μ in the core **is finite**;

The flux is not confined to the core. Not all the flux linking one of the windings links the other windings

Core losses are zero (Electrical conductivity of the core is zero)

Winding resistances is present

Real transformer:

Permeability μ in the core **is finite**;

The flux is not confined to the core. Not all the flux linking one of the windings links the other windings

Core losses are occurring

Winding resistances is present

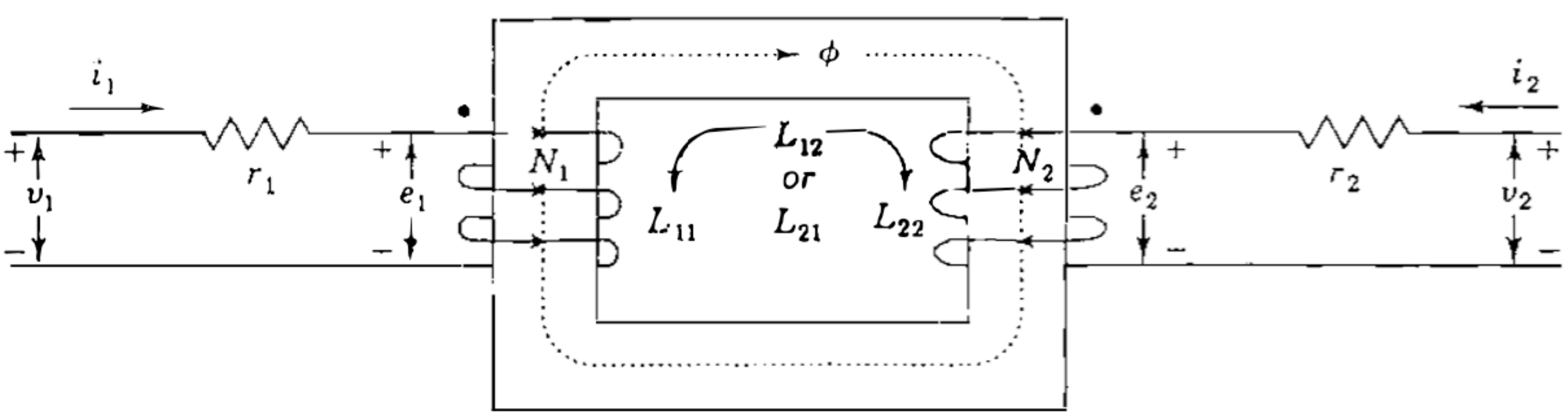
Magnetically coupled coils

In Fig. 1.3 the direction of current i_2 is chosen to produce flux (according to the right-hand rule) in the same sense as i_1 when both currents are either positive or negative. This choice gives positive coefficients in the equations which follow.

$\boldsymbol{X_{ab}}$ expresses:

- quantity X (flux or inductance)
- of coil a
- due to current i_b

Figure 1.3 (a):
Mutual flux due to currents i_1 and i_2



(a)

Magnetically coupled coils

In Fig. 1.3 (b) and (c) it is shown the effect of each of the two currents, respectively i_1 and i_2, acting alone.

$\boldsymbol{X_{ab}}$ expresses:

- quantity X (flux or inductance)
- of coil a
- due to current i_b

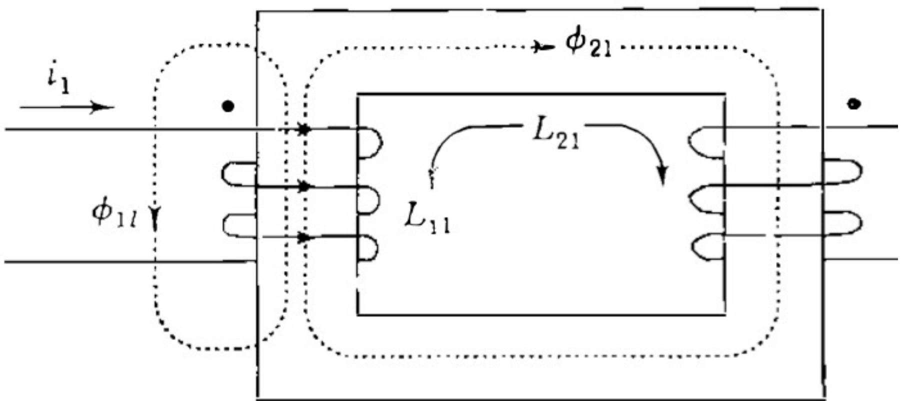


Figure 1.3 (b):
Leakage flux ϕ_{1l}
and mutual flux ϕ_{21}
due to $i_1$alone

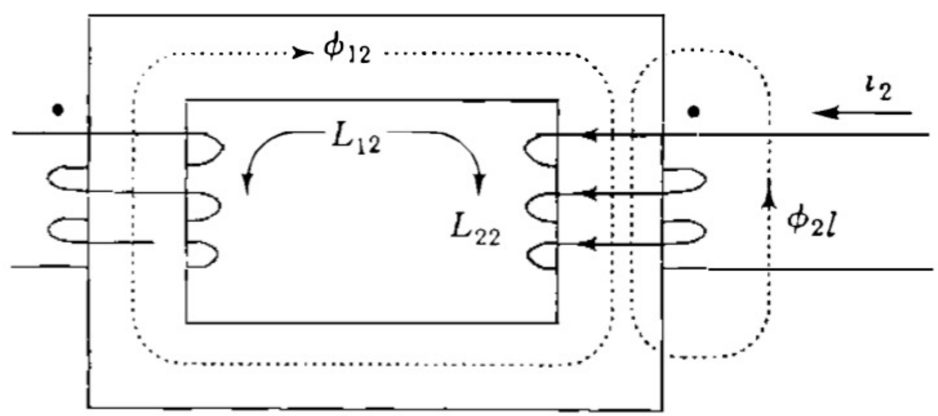
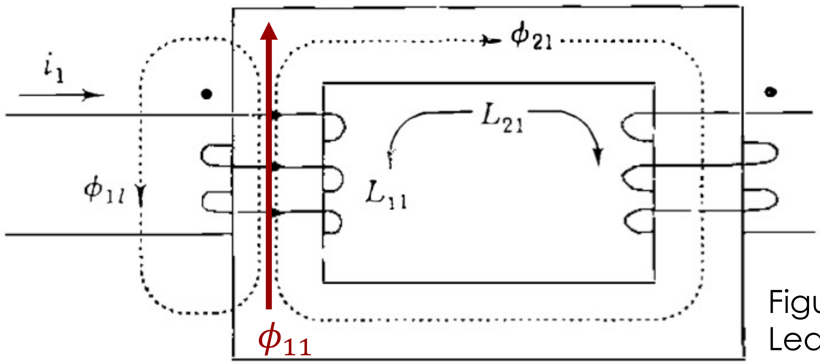


Figure 1.3 (c):
Leakage flux ϕ_{2l}
and mutual flux ϕ_{12}
due to i_2 alone

Magnetically coupled coils

i_1 acting alone



$$\phi_{11} = \phi_{21} + \phi_{1l}$$

Figure 1.3 (b):
Leakage flux ϕ_{1l} and mutual flux ϕ_{21} due to $i_1$alone

The current i_1 acting alone produces flux ϕ_{11}which has a mutual component ϕ_{21} linking both coils and a small leakage component ϕ_{1l} linking only coil one. The flux linkages of coil 1 due to current i_1 acting alone are given by:

$$\lambda_{11} = N_1\phi_{11} = L_{11}i_1 \tag{1.10}$$

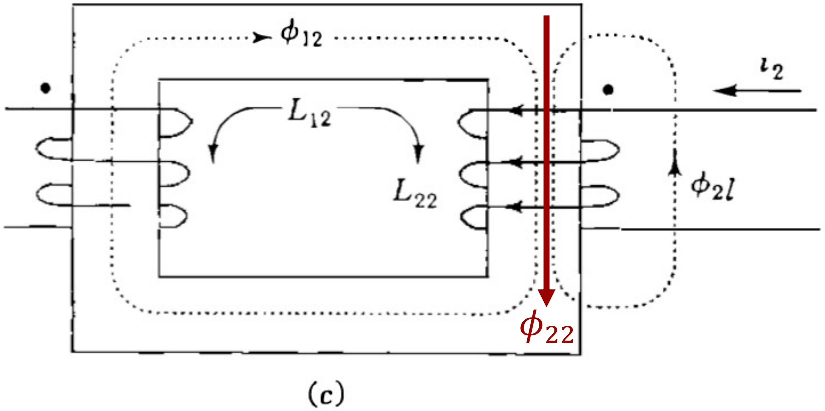
where N_1 is the number of turns and L_{11} is the self-inductance of coil 1.
Under the same condition of i_1 acting alone the flux linkages of coil 2 are given by:

$$\lambda_{21} = N_2\phi_{21} = L_{21}i_1 \tag{1.11}$$

where N_2 is the number of turns and L_{21} is the mutual-inductance between the coils.

Magnetically coupled coils

i_2 acting alone



$$\phi_{22} = \phi_{12} + \phi_{2l}$$

Figure 1.3 (c):
Leakage flux ϕ_{2l} and mutual flux ϕ_{12} due to i_2 alone

The current i_2 acting alone produces flux ϕ_{22} which has a mutual component ϕ_{12} linking both coils and a small leakage component ϕ_{2l} linking only coil one. The flux linkages of coil 2 due to current i_2 acting alone are given by:

$$\lambda_{22} = N_2\phi_{22} = L_{22}i_2 \tag{1.12}$$

where N_2 is the number of turns and L_{22} is the self-inductance of coil 2.
Under the same condition of i_2 acting alone the flux linkages of coil 1 are given by:

$$\lambda_{12} = N_1\phi_{12} = L_{12}i_2 \tag{1.13}$$

where N_1 is the number of turns and L_{12} is the mutual-inductance between the coils. Please note that mutual inductance is a single reciprocal property of the coils, and so $L_{12} = L_{21} > 0$ because of i_1 and i_2.

Magnetically coupled coils

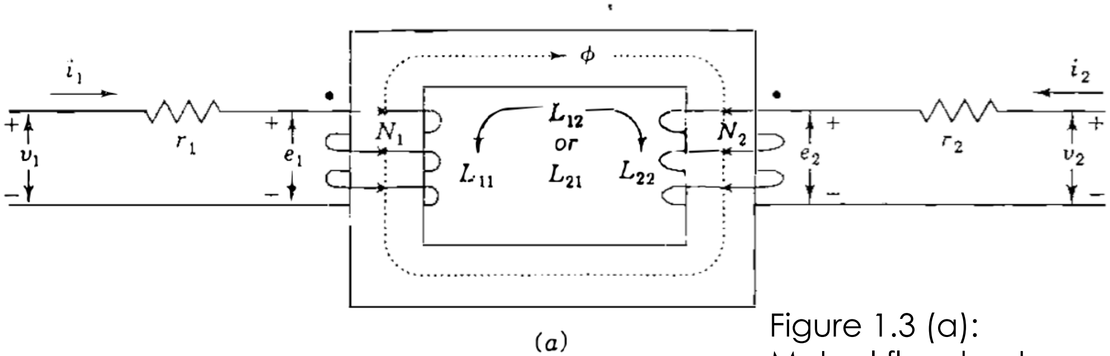


Figure 1.3 (a):
Mutual flux due to currents i_1 and i_2

When both currents act together, the flux linkages add (the transformer is supposed, for the moment, to be linear) to give:

$$\lambda_1 = \lambda_{11} + \lambda_{12} = L_{11}i_1 + L_{12}i_2 \qquad (1.14)$$

$$\lambda_2 = \lambda_{22} + \lambda_{21} = L_{21}i_1 + L_{22}i_2 \qquad (1.15)$$

It is now possible to rewrite Eq. (1.1), expressing Faraday's law, but considering a winding resistance greater than 0 :

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt} = r_1 i_1 + L_{11}\frac{di_1}{dt} + \mathrm{L}_{12}\frac{di_2}{dt} \qquad (1.16)$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt} = r_2 i_2 + L_{21}\frac{di_1}{dt} + \mathrm{L}_{22}\frac{di_2}{dt} \qquad (1.17)$$

where $N_1\frac{d\phi}{dt} = \frac{d\lambda}{dt}$, namely the flux linkages.

Magnetically coupled coils

The obtained equations are:

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt} = r_1 i_1 + L_{11}\frac{di_1}{dt} + \mathrm{L}_{12}\frac{di_2}{dt} \tag{1.16}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt} = r_2 i_2 + L_{21}\frac{di_1}{dt} + \mathrm{L}_{22}\frac{di_2}{dt} \tag{1.17}$$

These can be modified as follow:

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt} = r_1 i_1 + L_{11}\frac{di_1}{dt} - \mathrm{L}_{12}\frac{\mathrm{N}_1}{\mathrm{N}_2}\frac{di_1}{dt} + \mathrm{L}_{12}\frac{\mathrm{N}_1}{\mathrm{N}_2}\frac{di_1}{dt} + \mathrm{L}_{12}\frac{di_2}{dt} \tag{1.16}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt} = r_2 i_2 + \mathrm{L}_{22}\frac{di_2}{dt} - \mathrm{L}_{21}\frac{\mathrm{N}_2}{\mathrm{N}_1}\frac{di_2}{dt} + \mathrm{L}_{21}\frac{\mathrm{N}_2}{\mathrm{N}_1}\frac{di_2}{dt} + L_{21}\frac{di_1}{dt} \tag{1.17}$$

If we now consider $a = N_1/N_2$

$$v_1 = r_1 i_1 + \frac{d}{dt}[(L_{11} - L_{12}a)i_1 + L_{12}a i_1 + L_{12} i_2]$$
$$v_2 = r_2 i_2 + \frac{d}{dt}[(L_{22} - L_{21}/a)i_2 + L_{21}/a i_2 + L_{12}\, i_1]$$

Magnetically coupled coils

$$v_1 = r_1 i_1 + \frac{d}{dt}[(L_{11} - L_{12}a)i_1 + L_{12}ai_1 + L_{12}i_2]$$
$$v_2 = r_2 i_2 + \frac{d}{dt}[(L_{22} - L_{21}/a)i_2 + L_{12}/ai_2 + L_{12}i_1]$$

The quantity $(L_{11} - aL_{12})$ is the **leakage inductance** $\boldsymbol{L_{1l}}$ (by knowing that $\phi_{11} = \phi_{21} + \phi_{1l}$) and that aL_{12} is the **magnetizing inductance** associated with the mutual flux ϕ_{21} linking the coils due to i_1

$(L_{11} - aL_{21}) \ = L_{1l} =$ Leakage inductance 1
$(L_{22} - L_{21}/a) = L_{2l} =$ Leakage inductance 2

$$v_1 = r_1 i_1 + \frac{d}{dt}\left[L_{1l}i_1 + \ N_1\left(\frac{L_{12}}{N_2}i_1 + \frac{L_{21}}{N_1}i_2\right)\right]$$
$$v_2 = r_2 i_2 + \frac{d}{dt}\left[L_{2l}i_2 + N_2\left(\frac{L_{12}}{N_1}i_2 + \frac{L_{21}}{N_2}i_1\right)\right]$$

Note that $L_{21} = L_{12} = \frac{\lambda_{12}}{i_2} = \frac{\lambda_{21}}{i_1} = \frac{N_2\phi_{21}}{i_1} = \frac{N_1\phi_{12}}{i_2}$ from (1.10) and (1.13)

Magnetically coupled coils

$$v_1 = r_1 i_1 + \frac{d}{dt}\left[L_{1l} i_1 + \boxed{N_1\left(\frac{L_{12}}{N_2} i_1 + \frac{L_{21}}{N_1} i_2\right)}\right]$$

$$v_2 = r_2 i_2 + \frac{d}{dt}\left[L_{2l} i_2 + N_2\left(\frac{L_{12}}{N_1} i_2 + \frac{L_{21}}{N_2} i_1\right)\right]$$ by referring this to the primary we have:

$$v_2 a = r_2 i_2 a + \frac{d}{dt}\left[L_{2l} i_2 a + N_2 a\left(\frac{L_{12}}{N_1} i_2 + \frac{L_{21}}{N_2} i_1\right)\right]$$

$$v_2 a = r_2 i_2 \frac{N_1}{N_2}\frac{N_2}{N_1}\frac{N_1}{N_2} + \frac{d}{dt}\left[L_{2l} i_2 \frac{N_1}{N_2}\frac{N_2}{N_1}\frac{N_1}{N_2} + N_2 \frac{N_1}{N_2}\left(\frac{L_{12}}{N_1} i_2 + \frac{L_{21}}{N_2} i_1\right)\right]$$

$$v_2 a = r_2 i_2 \frac{N_1}{N_2}\frac{N_2}{N_1}\frac{N_1}{N_2} + \frac{d}{dt}\left[L_{2l} i_2 \frac{N_1}{N_2}\frac{N_2}{N_1}\frac{N_1}{N_2} + N_2 \frac{N_1}{N_2}\left(\frac{L_{12}}{N_1} i_2 + \frac{L_{21}}{N_2} i_1\right)\right]$$

$$v_2 a = (a^2 r_2)\frac{i_2}{a} + \frac{d}{dt}\left[(a^2 L_{2l})\frac{i_2}{a} + \boxed{N_1\left(\frac{L_{12}}{N_1} i_2 + \frac{L_{21}}{N_2} i_1\right)}\right]$$

Now, the terms in the red squares, common to both equations, are equal to:

$$N_1\left(\frac{L_{12}}{N_1} i_2 + \frac{L_{21}}{N_2} i_1\right) = aL_{21}(i_1 + i_2/a)$$

Magnetically coupled coils

$$v_1 = r_1 i_1 + \frac{d}{dt}\left[L_{1l} i_1 + \boxed{N_1\left(\frac{L_{12}}{N_2} i_1 + \frac{L_{21}}{N_1} i_2\right)}\right]$$

$$v_2 a = (a^2 r_2)\frac{i_2}{a} + \frac{d}{dt}\left[(a^2 L_{2l})\frac{i_2}{a} + \boxed{N_1\left(\frac{L_{12}}{N_1} i_2 + \frac{L_{21}}{N_2} i_1\right)}\right]$$

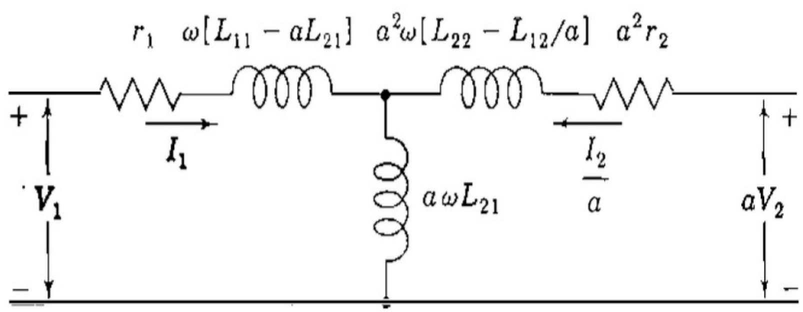
The terms in the red squares, common to both equations, are equal to:

$$N_1\left(\frac{L_{12}}{N_1} i_2 + \frac{L_{21}}{N_2} i_1\right) = aL_{21}(i_1 + i_2/a)$$

$$v_1 = r_1 i_1 + \frac{d}{dt}[L_{1l} i_1 + aL_{21}(i_1 + i_2/a)\ ]$$

$$v_2 a = (a^2 r_2)\frac{i_2}{a} + \frac{d}{dt}\left[(a^2 L_{2l})\frac{i_2}{a} + aL_{21}(i_1 + i_2/a)\ \right]$$

$$v_2' = r_2' i_2' + \frac{d}{dt}[L_{2l}' i_2' + aL_{21}(i_1 + i_2/a)\ ]$$



Note that the notation of the equations refers to time-domain quantities while the one in the figure refers to frequency-domain quantities (i.e. phasors)

Magnetically coupled coils

An important equivalent circuit, **referred to the primary winding**, for the mutually coupled coils is shown in Fig. 1.4.
All the quantities of the secondary winding are referred to the primary in analogy with Eq. (1.8). It is possible, by writing Kirchhoff's voltage equation around the path of each of the currents I_1 and $\frac{I_2}{a}$, to obtain the two fundamental Eq.(1.18) and (1.19).

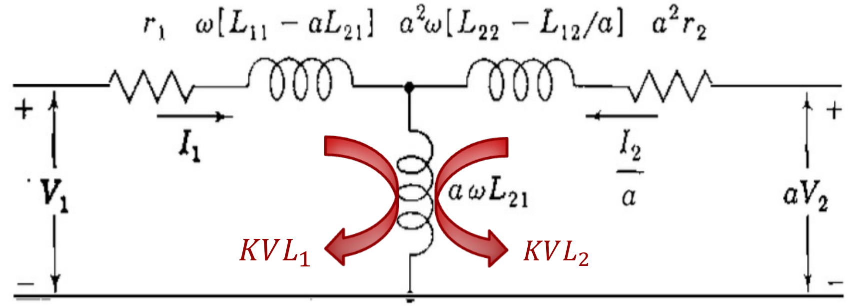


Figure 1.4: An AC equivalent circuit with secondary current and voltage redefined and a = N1/N2.

KVL_1:

$$\bar{V}_1 = \bar{I}_1 r_1 + \bar{I}_1(j\omega L_{11}) - a\bar{I}_1(j\omega L_{21}) + a(j\omega L_{21})\left(\bar{I}_1 + {}^{\bar{I}_2}/_{a}\right)$$

$$\boxed{\bar{V}_1 = \bar{I}_1 r_1 + \bar{I}_1(j\omega L_{11}) + \bar{I}_2(j\omega L_{21})} \rightarrow \text{Eq. (1.18)}$$

KVL_2:

$$a\bar{V}_2 = \left(\frac{\bar{I}_2}{a}\right)a^2 r_2 + \left(\frac{\bar{I}_2}{a}\right)a^2(j\omega L_{22}) - \left(\frac{\bar{I}_2}{a}\right)a^2\left(j\omega\frac{L_{12}}{a}\right) + a(j\omega L_{21})\left(\bar{I}_1 + \frac{\bar{I}_2}{a}\right)$$

$$\bar{V}_2 = \bar{I}_2 r_2 + \bar{I}_2(j\omega L_{22}) - \bar{I}_2\left(j\omega\frac{L_{12}}{a}\right) + \bar{I}_2\left(j\omega\frac{L_{21}}{a}\right) + \bar{I}_1(j\omega L_{21})$$

$$\boxed{\bar{V}_2 = \bar{I}_2 r_2 + \bar{I}_2(j\omega L_{22}) + \bar{I}_1(j\omega L_{21})} \rightarrow \text{Eq. (1.19)}$$

Magnetically coupled coils

Summary

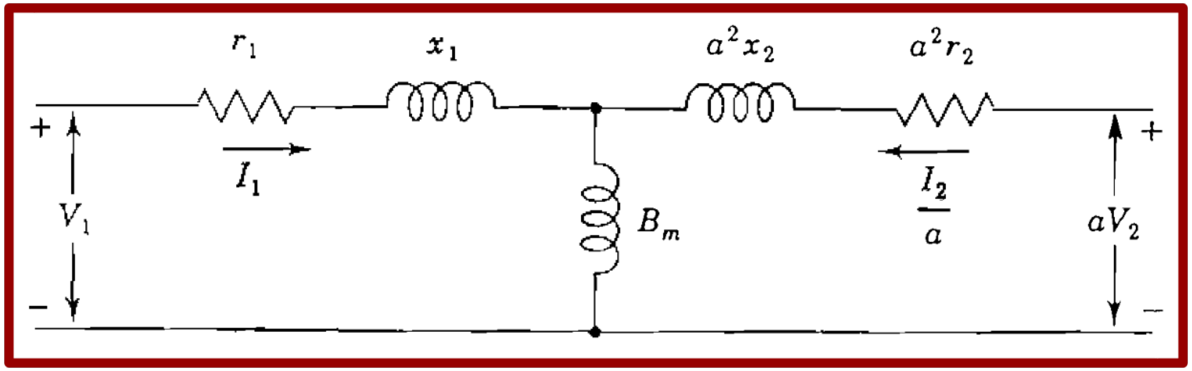


Figure 1.5 The **equivalent circuit** of Fig. 1.4 with inductance parameters renamed as follow:

$x_1 = \omega(L_{11} - aL_{21}) \; = \omega L_{1l}$ = Leakage inductance 1 *

$x_2 = \omega(L_{22} - L_{21}/a) = \omega L_{2l}$ = Leakage inductance 2 *

$B_m = (a\omega L_{21})^{-1}$ = Shunt magnetizing susceptance*

Hypothesis:

- Flux varies **sinusoidally** in the core
- **Real transformer** but neglecting core losses
- Equivalent circuit referred to the **primary** winding

Main equations:

$$\boxed{\begin{aligned}\bar{V}_1 &= (r_1 + j\omega L_{11})\bar{I}_1 + (j\omega L_{12})\bar{I}_2 \\ \bar{V}_2 &= (j\omega L_{21})\bar{I}_1 + (r_2 + j\omega L_{22})\bar{I}_2\end{aligned}} \qquad \begin{aligned}(1.18)\\(1.19)\end{aligned}$$

* It is possible to prove that the quantity $(L_{11} - aL_{21})$ is the leakage inductance L_{1l} by knowing that $\phi_{11} = \phi_{21} + \phi_{1l}$ and that aL_{21} is a magnetizing inductance associated with the mutual flux ϕ_{21} linking the coils due to i_1 since:

$$aL_{21} = \frac{N_1}{N_2}\frac{N_2\phi_{21}}{i_1} = \frac{N_1}{i_1}\phi_{21} \qquad (1.20)$$

Outline

The ideal transformer model

Magnetically coupled coils

The equivalent circuit of a single-phase transformer

Simplified equivalent circuit of single phase transformer

Three-phase transformers

The equivalent circuit of a single-phase transformer

A further step has to be done to match the physical characteristics of the practical transformer. In fact, the previous equivalent circuit presents **three main deficiencies**:

- It does not reflect any current or **voltage transformation**,
- It does not provide for **electrical isolation** of the primary from the secondary
- It does not account for the **core losses**.

Observation#1:

If a sinusoidal voltage is applied to the primary winding of a transformer with the secondary winding open, a small current I_E called the "exciting current flows". This current I_E is composed by the current flowing through the magnetizing susceptance B_m, called magnetizing current and a **much smaller component which accounts for losses.**

$$\bar{I}_E = \bar{I}_{EM} + \bar{I}_{EL} \tag{1.21}$$

Exciting current=Magnetising current + Losses

Please note that so far we have been neglected the core losses by stating: $\bar{I}_E \cong \bar{I}_{EM}$.

The equivalent circuit of a single-phase transformer

Observation#2:
The core losses occur due to two different phenomena:

- **Hysteresis loss:** cyclic changes of the direction of the flux in the iron require energy which is dissipated as heat. These losses are reduced by the use of certain high grades of alloy steel for the core

- **Eddy-current loss:** circulating currents are induced in the iron due to the changing flux dissipating a power of $|I|^2R$. Eddy-cur rent loss is reduced by building up the core with laminated sheets of steel.

Observation#3:

The component of $\bar{I}_{EL}$ that account for the losses leads the magnetizing current $\bar{I}_{EM}$ by 90°. In the equivalent circuit, $\bar{I}_E$ is taken fully into account by a conductance G_C ($\bar{I}_{EL}$) in parallel with the magnetizing susceptance B_m ($\bar{I}_{EM}$).

The equivalent circuit of a single-phase transformer

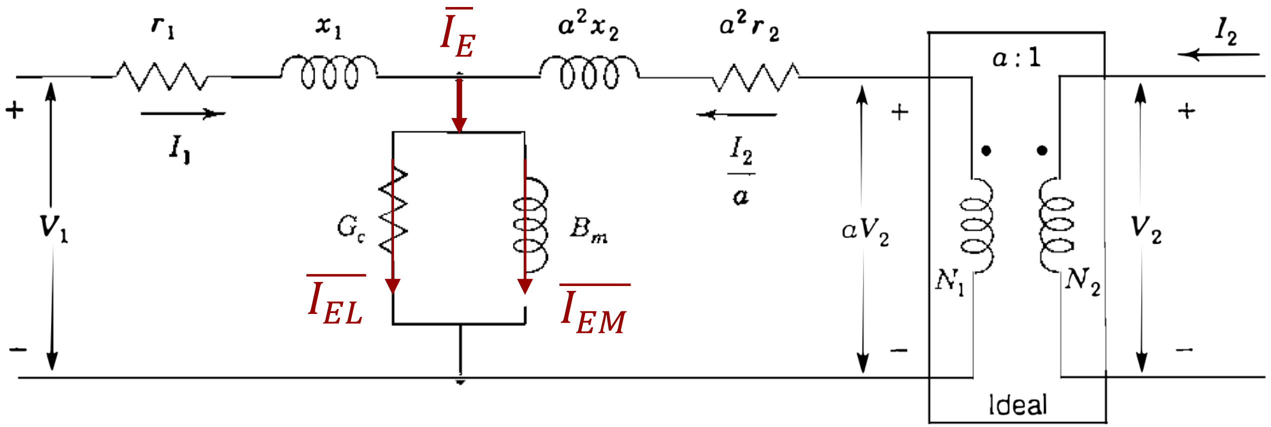


Figure 1.6
Equivalent circuit for a single-phase transformer with an ideal transformer of turns ratio a = N_1/N_2.

Main components:

- An ideal transformer provides voltage and current transformation and electrical isolation of the primary from the secondary. Its characteristics are described by Eqs. (1.3) and (1.6).
- The equivalent circuit presented in Fig. 1.5 takes into account a finite permeability of the core (i.e. magnetizing currents $\overline{I}_{EM}$), flux leakages and winding resistances.
- The conductance G_c represents the core losses

The equivalent circuit of a single-phase transformer

Starting from Fig. (1.6) we can further simplify the problem by neglecting the exciting current and omitting the ideal transformer. In this case, all impedances and voltages in the part of the circuit connected to the secondary terminals must now be referred to the primary side. The resulting equivalent circuit is shown in Fig. (1.7) where the parameters R_1 and $X_1$are defined as follow:

$$R_1 = r_1 + a^2 r_2 \qquad X_1 = x_1 + a^2 x_2 \tag{1.22}$$

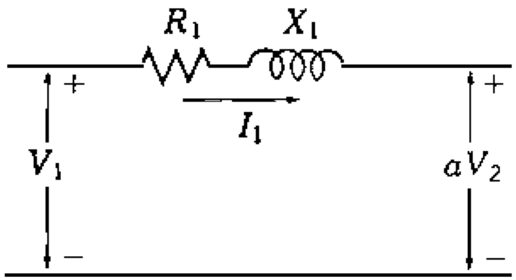


Figure 1.7
Transformer equivalent circuit with magnetizing current neglected

Voltage regulation is defined as the difference between the voltage magnitude at the load terminals of the transformer at full load and at no-load in percent of full-load voltage with input voltage held constant. In the form of an equation:

$$\text{Percent regulation} = \frac{|V_{2,NL}| - |V_{2,FL}|}{|V_{2,FL}|} \times 100 \tag{1.23}$$

Outline

The ideal transformer model

Magnetically coupled coils

The equivalent circuit of a single-phase transformer

Simplified equivalent circuit of single phase transformer

Three-phase transformers

Simplified equivalent circuit of a single-phase transformer

Hypothesis:

- Flux varies **sinusoidally** in the core
- **Real transformer, considering core losses**
- Equivalent circuit referred to the **primary** winding

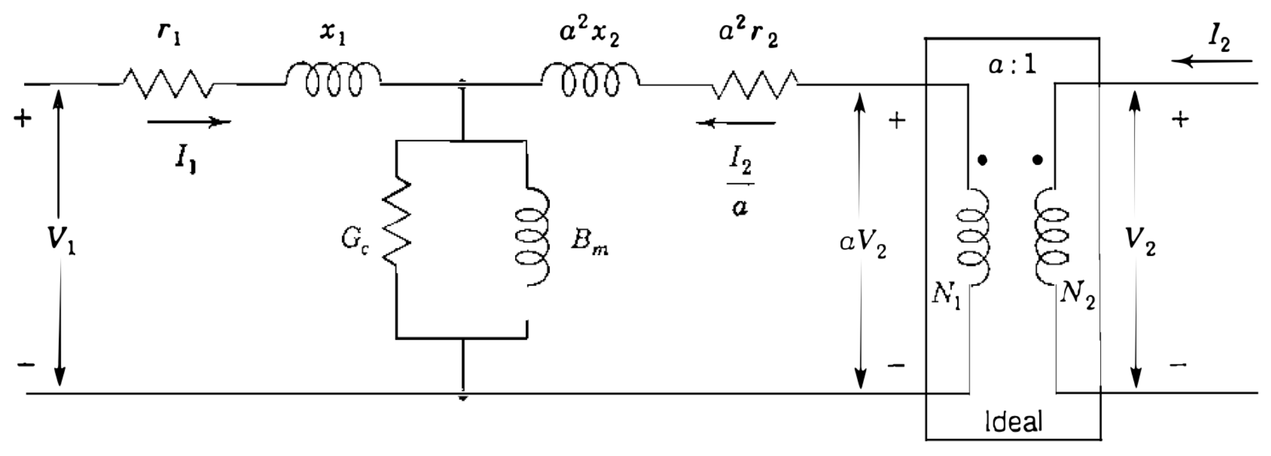


Figure 1.8: **Equivalent circuit** for a single-phase transformer with an ideal transformer with the following parameters:

$\boldsymbol{a}$ = turns ratio a
$\boldsymbol{r_1}$ = winding resistance 1
$\boldsymbol{r_2}$ = winding resistance 2

$\boldsymbol{x_1} = \omega L_{1l}$ = Leakage inductance 1
$\boldsymbol{x_2} = \omega L_{2l}$ = Leakage inductance 2
$\boldsymbol{B_m}$ = Shunt magnetizing susceptance

Simplified equivalent circuit of a single-phase transformer

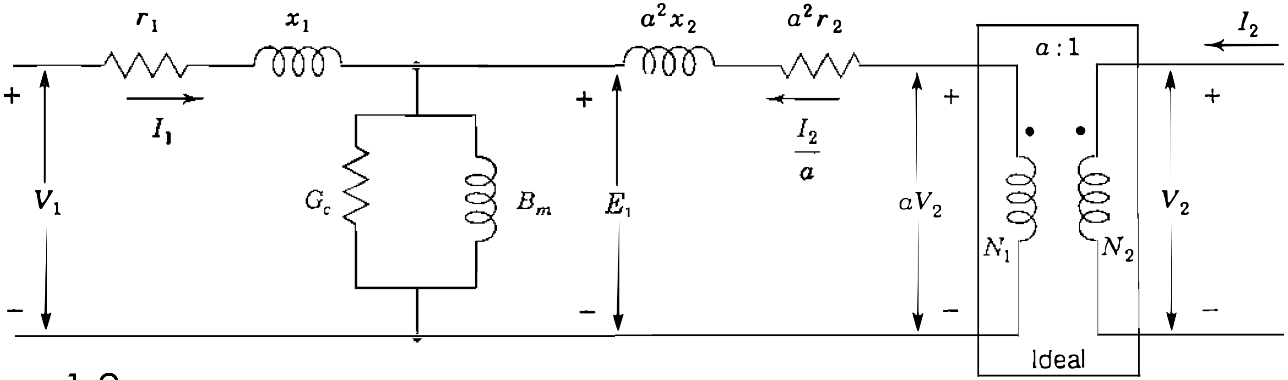


Figure 1.9:
Equivalent circuit for a single-phase transformer with an ideal transformer

Consideration #1:
The voltage drop across $\bar{z}_1 = r_1 + jx_1$ is very small

Consequences:

- Under this condition $\bar{E}_1 \cong \bar{V}_1$
- It is possible to simplify the circuit by moving the impedance $\bar{z}_1$ on the right so that it results in series with a^2x_2 and $a^2\mathrm{r}_2$
- The new impedance $\bar{Z}_{cc}$ is calculated as:

$$\bar{Z}_{cc} = \bar{z}_1 + j(a^2x_2) + a^2r_2 \qquad (1.24)$$

Simplified equivalent circuit of a single-phase transformer

The new equivalent circuit, that lies on the hypothesis of small voltage drop across the impedance $z_1$is shown in Fig. (1.10)

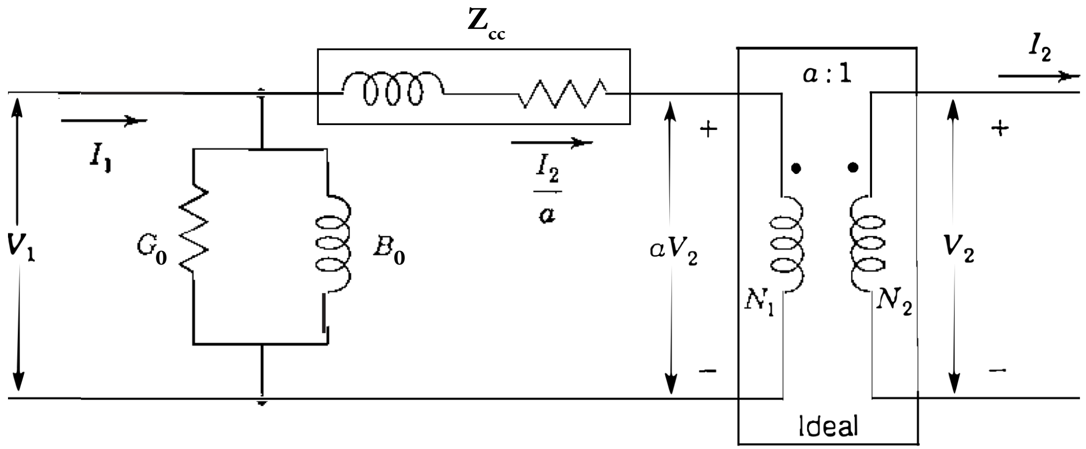


Figure 1.10: **Equivalent circuit** for a single-phase transformer with an ideal transformer (Note that $G_c$and B_m have been re-named into G_0 and B_0 and the sign of I_2 has been changed in order to get positive component in the following equations).

This equivalent circuit is particularly important because **its parameters can be determined by a short circuit test and an open circuit test**. Indeed, if the secondary of the transformer is open ($I_2 = 0$), the current I_1 is flowing just through $\bar{Y}_0 = G_0 + jB_0$ and if the secondary is short-circuited ($V_2 = 0$), the current I_1 is mainly flowing through $\bar{Z}_{cc} = \bar{z}_1 + j(a^2 x_2) + a^2 r_2$

Simplified equivalent circuit of a single-phase transformer

The equations ruling the equivalent circuit of Fig.(1.11) are the following:

$$\boxed{\begin{aligned} \bar{V}_1 &= a\bar{V}_2 + \bar{Z}_{cc} \cdot \frac{\bar{I}_2}{a} \qquad (1.25) \\ \bar{I}_1 &= \bar{Y}_0(a\bar{V}_2) + (\bar{Y}_0\bar{Z}_{cc} + 1) \cdot \frac{\bar{I}_2}{a} \qquad (1.26) \end{aligned}}$$

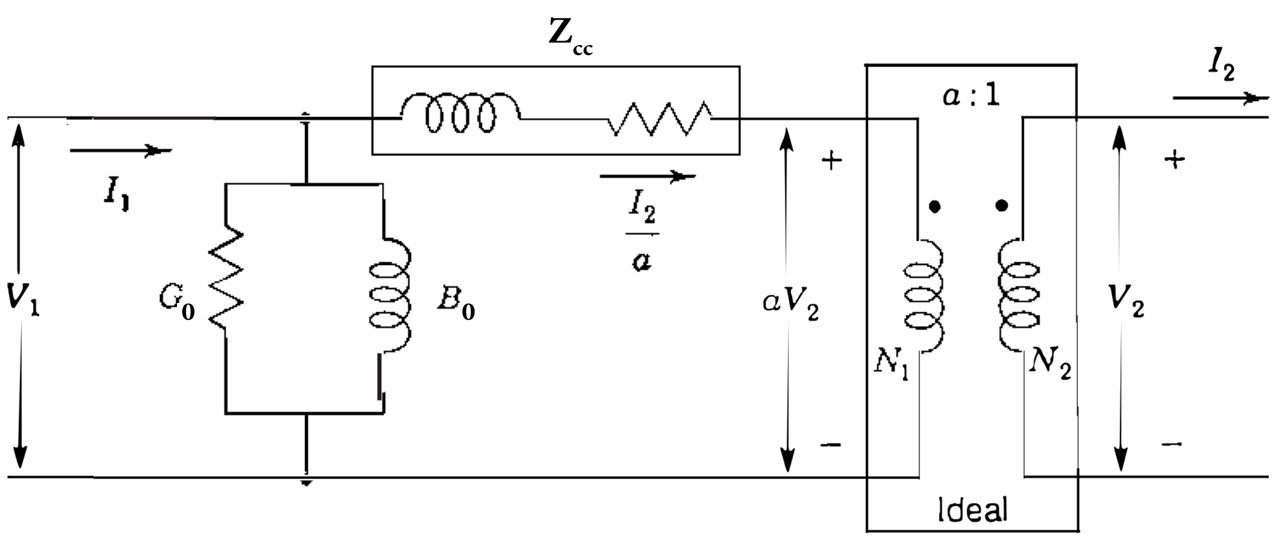


Figure 1.11

Outline

The ideal transformer model

Magnetically coupled coils

The equivalent circuit of a single-phase transformer

Simplified equivalent circuit of single-phase transformer

Three-phase transformers

Three-phase transformers

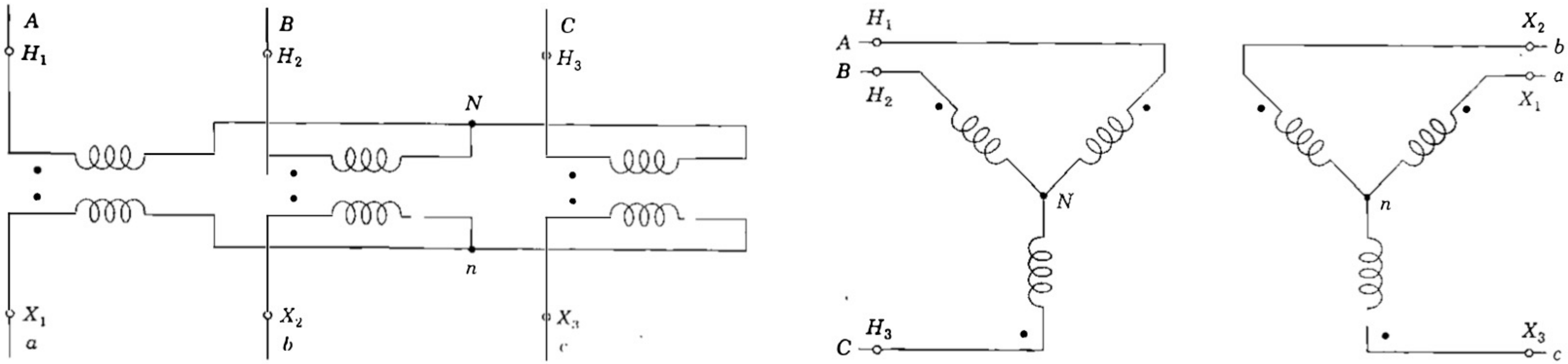
Three identical single-phase transformers may be connected so that the three windings of one voltage rating are Δ-connected or Y-connected
There are many possible connections such as:

- Y-Δ
- Δ-Y
- Y-Y
- Δ-Δ

Instead of using three identical single-phase transformers, a more usual unit is a three phase transformer where all three phases are on the same iron structure

- The **three-phase unit** has the advantage of **requiring less iron** to form the core, and is therefore more economical than three single-phase units and occupies less space.
- **Three single-phase units** have the advantage of **replacement of only one unit of the three-phase bank in case of failure** rather than losing the whole three-phase bank.
- If a failure occurs in a Δ - Δ bank composed of three separate units, one of the single-phase transformers can be removed and the remaining two will still operate as a three-phase transformer at a reduced kVA. Such an operation is called **open delta**.

Three-phase transformers



(a) Y-Y connection diagram

(b) Alternate form of connection diagram

Figure 1.12
Wiring diagrams for Y-Y transformer.

In the following slides:

- Capital letters A, B, and C to identify the phases of the high-voltage windings and
- Lowercase letters a, b, and c for the low-voltage windings.
- The high-voltage terminals of three-phase transformers are marked H_1, H_2, H_3
- The low-voltage terminals are marked X_1, X_2, X_3.

Three-phase transformers

Different connections

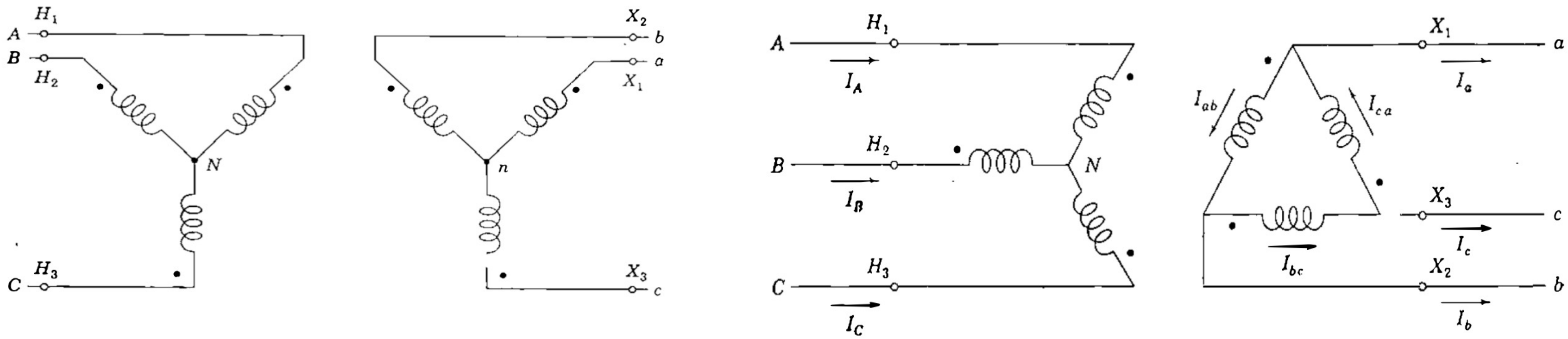


Figure 1.13:
Wiring diagram two three-phase transformers connected Y-Y (left) and Y-Δ (right)

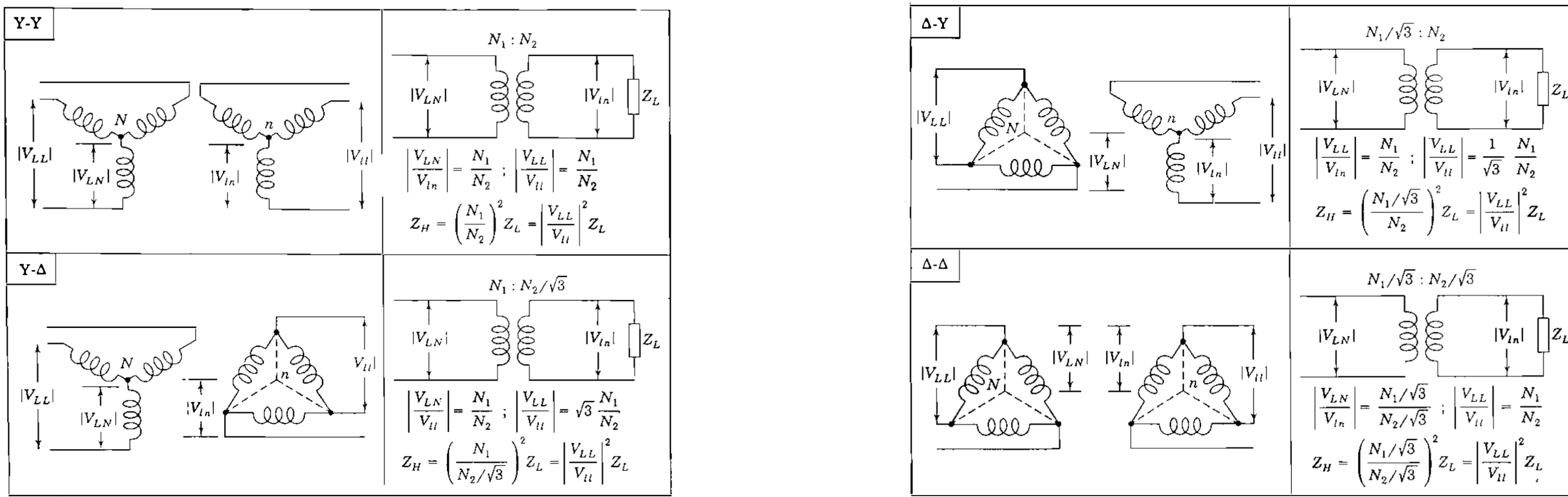
Since it is possible to connect the windings in different configurations, it is important to understand how the **magnetic links** between high and low voltage sides of the transformer are changing.

- In Y-Y transformers the markings are such that line-to-neutral HV windings are always magnetically linked with the line-to-neutral LV windings
- In Δ-Δ transformers the markings are such that line-to-line HV windings are always magnetically linked with the line-to-line LV windings
- In Y-Δ transformers the markings are such that line-to-neutral HV windings are always magnetically linked with the line-to-line LV windings
- In Δ-Y transformers the markings are such that line-to-line HV windings are always magnetically linked with the line-to-phase LV windings

In these last two cases the **effective ratio** will not be equal to the turns ratio and a **shift phase** will occur.

Three-phase transformers

Effective ratio



The **effective ratio** $\mathbf{r}$ can be defined as the ratio $\mathbf{r} = |\bar{V}_{LL}/\bar{V}_{ll}|$ **between the line-to-line voltages.** This ratio is not equal to the **turns ratio** $\mathbf{a} = \boldsymbol{N_1/N_2}$ but also a function of the geometry of the system.

- In Y-Y or Δ-Δ transformers the markings are such that line-to-line voltage ratio is given by the turns ratio a.
- In Y-Δ transformers the markings are such that the turns ratio $\mathbf{a}$ express the ratio between the line-to-neutral voltage of the high voltage side and the line-to-line voltage of the low voltage side. The effective ratio can be calculated by the following equation:

$$\mathrm{r} = \left|\frac{\bar{V}_{LL}}{\bar{V}_{ll}}\right| = \left|\frac{\sqrt{3}\ \bar{V}_{LN}}{\bar{V}_{ll}}\right| = \sqrt{3}\left|\frac{\bar{V}_{LN}}{\bar{V}_{ll}}\right| = \sqrt{3}\frac{N_1}{N_2} = \sqrt{3}a$$

- In the same way, to transfer impedance from the voltage level on one side of a three-phase transformer to the voltage level on the other, the multiplying factor is the effective ratio and not the turns ratio

Three-phase transformers

Phase Shift

Let's consider a three-phase transformer connected Y-Δ where Y side is the high-voltage side. As previously discussed, the markings are such that line-to-neutral HV windings are **magnetically linked** with the line-to-line LV windings. Therefore, $\bar{V}_{AN}$ is always in phase with $\bar{V}_{ab}$.

As a result, the line-to-neutral voltage phase $\angle V_{an}$ is shifted by 30° in respect to $\angle V_{AN}$ as visible from Fig. 1.14.

Fig.1.14 shows that $\bar{V}_{\mathrm{AN}} = \left(\frac{N_1}{N_2}\right)\bar{V}_{ab}$, i.e. the line-to-neutral primary voltage is in phase with the line-to-line secondary voltage. By looking at the phasor geometry we obtain:

$$\bar{V}_{AN} = \frac{N_1}{N_2}\sqrt{3}\,\bar{V}_{an}\,\angle 30° \qquad (1.27)$$

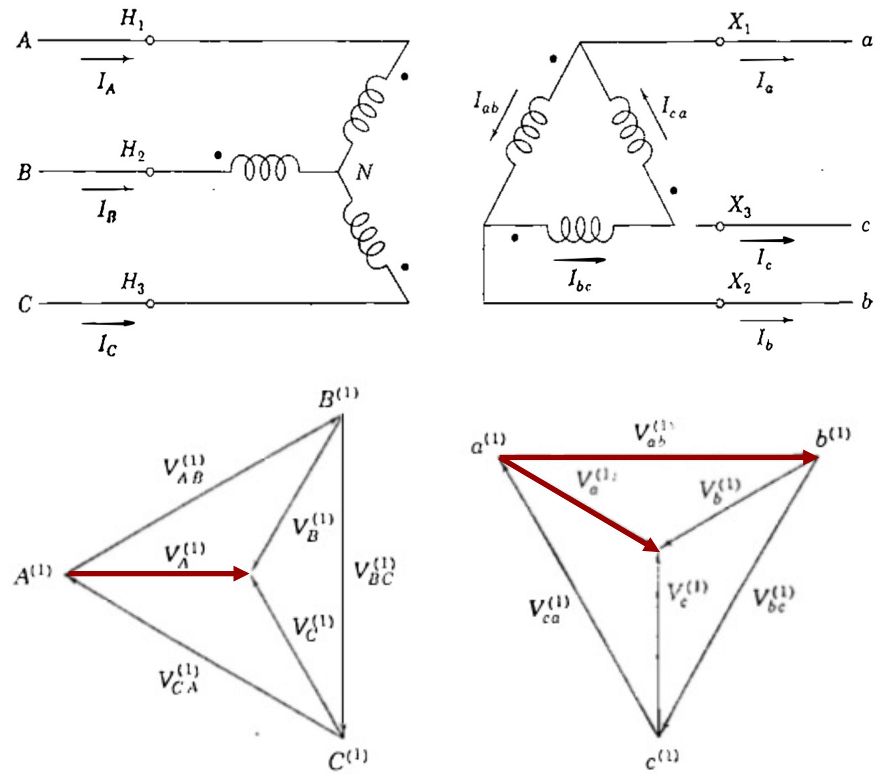


Figure 1.14:
Wiring diagram and voltage phasors for a three-phase transformer connected Y-Δ where Y side is the high-voltage side.

Three-phase transformers

- Transformer impedance and magnetizing currents are handled separately from **the phase shift**, which **can be represented by an ideal transformer** as in Fig. (1.15) .
- Usually, the **high-voltage** winding in a Y-Δ transformer is **Y-connected** to reduce insulation costs for a given step-up voltage since this connection takes advantage of the fact that the voltage transformation is then $\sqrt{3}(N_1/N_2)$.

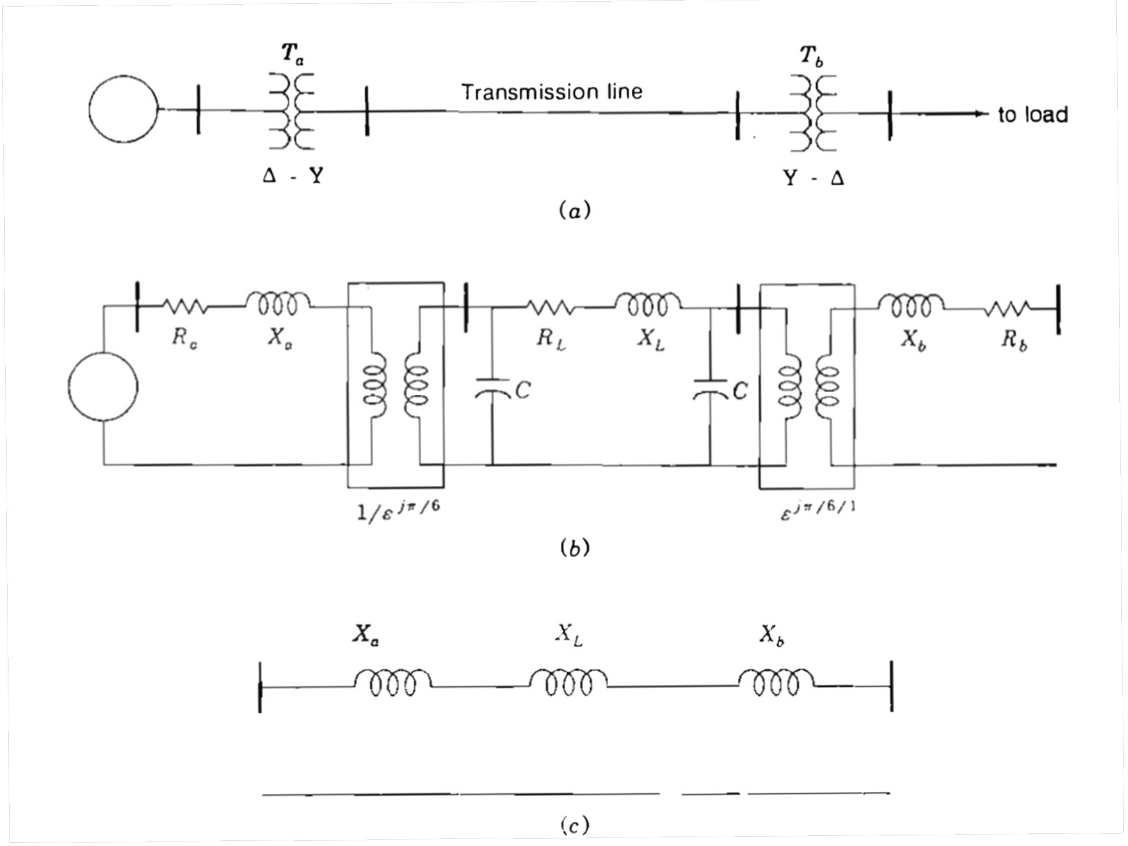


Figure (1.15)

(a) Single-line diagram;

(b) Per-phase equivalent circuits with parameters in per unit;

(c) Per-phase equivalent circuit with resistance, capacitance, and ideal transformers neglected.

Tap changing and regulating transformers

- Transformers which provide a small adjustment of voltage magnitude (in the range of **± 10%**)and others which shift the phase angle of the line voltages are important components of a power system.

- A **type of transformer** designed for **small adjustments of voltage** rather than large changes in voltage levels is called a **regulating transformer**.

- Almost all transformers provide taps on windings to adjust the ratio of transformation by changing taps when the transformer is de-energized but **a change in tap can be made while the transformer is energized, and such transformers are called load-tap-changing (LTC) transformers or tap-changing-under-load (TCUL) transformers.**